Parallel and Perpendicular Lines

Geometry
Chapter 3

Geometry 3

- This Slideshow was developed to accompany the textbook
  - *Larson Geometry*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

3.1 Identify Pairs of Lines and Angles
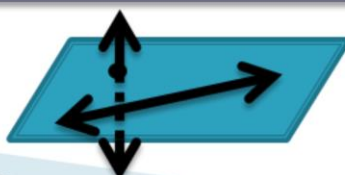
Parallel Lines ||

Lines that do NOT intersect and are coplanar

Lines go in the same direction

Skew Lines

Lines that do NOT intersect and are on different planes

Lines go in different directions

3

3.1 Identify Pairs of Lines and Angles

▸ Name the lines through point *H* that appear skew to $\overleftrightarrow{CD}$

▸ Name the lines containing point *H* that appear parallel to $\overleftrightarrow{CD}$

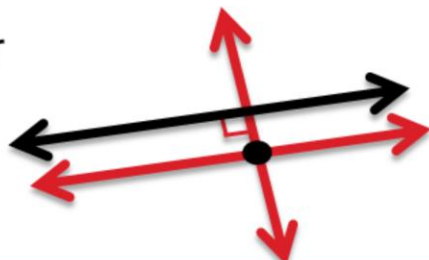▸ Name a plane that is parallel to plane *CDE* and contains point *H*

AH, EH
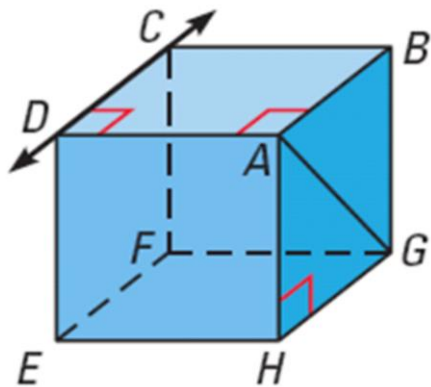
GH

BGH

3.1 Identify Pairs of Lines and Angles

▸ In a plane, two lines are either
  ◦ Parallel
  ◦ Intersect

Parallel Postulate

If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

3.1 Identify Pairs of Lines and Angles

Transversal

Line that intersects two coplanar lines
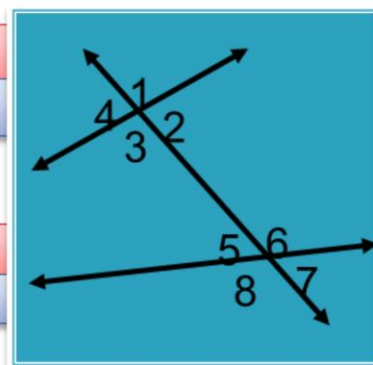
Interior ∠

angles that are between the lines

∠2, ∠3, ∠5, ∠6

Exterior ∠

angles that are outside of the lines

∠1, ∠4, ∠7, ∠8

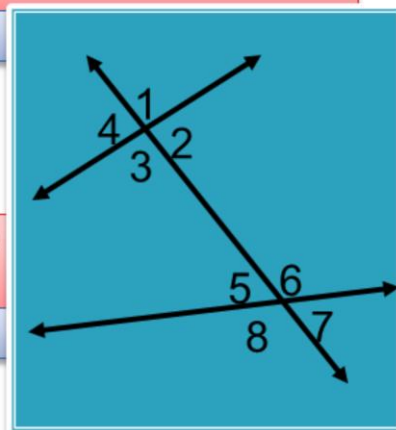# 3.1 Identify Pairs of Lines and Angles

Alternate interior angles

interior angles on opposite sides of the transversal

$\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 6$

Alternate exterior angles

exterior angles on opposite sides of the transversal

$\angle 1$ and $\angle 8$, $\angle 4$ and $\angle 7$

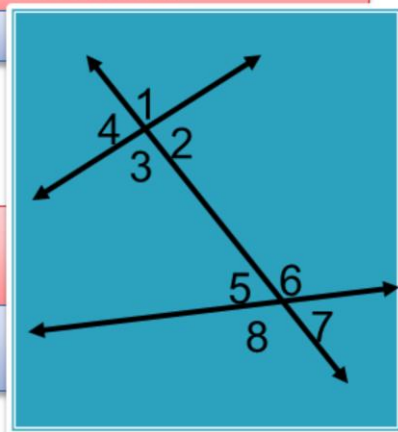# 3.1 Identify Pairs of Lines and Angles

Consecutive interior angles

interior angles on the same side of the transversal

$\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 5$

Corresponding angles

angles on the same location relative to the transversal
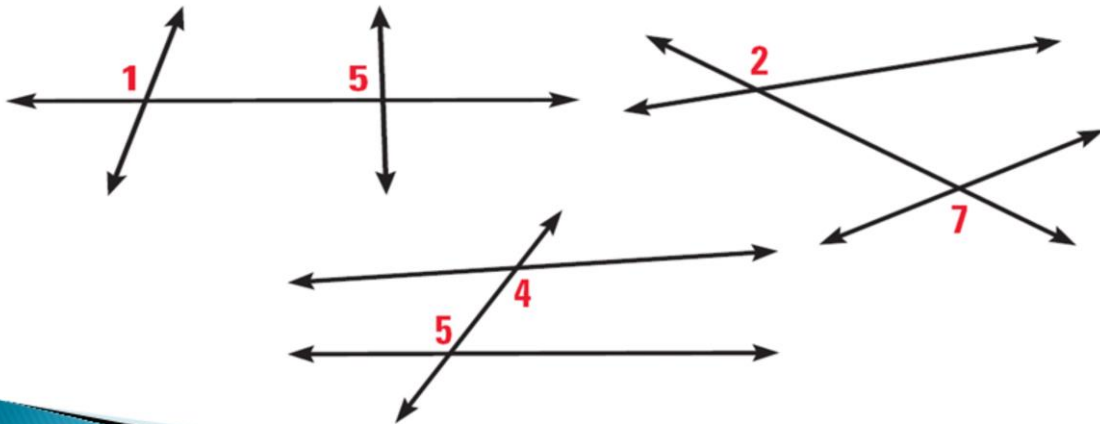
$\angle 1$ and $\angle 6$, $\angle 2$ and $\angle 7$,
$\angle 3$ and $\angle 8$, $\angle 4$ and $\angle 5$

3.1 Identify Pairs of Lines and Angles

▸ Classify the pair of numbered angles

150 #4–42 even, 45–49 all = 25 total

Corresponding
Alternate Exterior
Alternate Interior

Answers and Quiz

- 3.1 Answers

- 3.1 Quiz

3.2 Use Parallel Lines and Transversals

- Draw parallel lines on a piece of notebook paper, then draw a transversal.

- Use the protractor to measure all the angles.

- What types of angles are congruent?
  - (*corresponding, alt interior, alt exterior*)
- How are consecutive interior angles related?
  - (*supplementary*)

3.2 Use Parallel Lines and Transversals

Corresponding Angles Postulate

If 2 || lines are cut by trans., then the corrs ∠ are ≅

Alternate Interior Angles Theorem

If 2 || lines are cut by trans., then the alt int ∠ are ≅

Alternate Exterior Angles Theorem

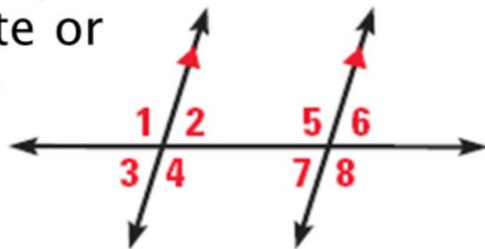If 2 || lines are cut by trans., then the alt ext ∠ are ≅

Consecutive Interior Angles Theorem

If 2 || lines are cut by trans., then the cons int ∠ are supp.

3.2 Use Parallel Lines and Transversals

▸ If m∠1 = 105°, find m∠4, m∠5, and m∠8. Tell which postulate or theorem you use in each case

▸ If m∠3 = 68° and m∠8 = (2x + 4)°, what is the value of x?

m∠4 = 105; vertical angles are congruent
m∠5 = 105; corresponding angles postulate
m∠8 = 105; alt ext angles theorem

m∠3 = m∠2
m∠8 = m∠5
∠2 and ∠5 are cons int angles and are supp
m∠2 + m∠5 = 180
m∠3 + m∠8 = 180
68 + 2x + 4 = 180
2x + 72 = 180
2x = 108
x = 54

3.2 Use Parallel Lines and Transversals

▸ Prove that if 2 || lines are cut by a trans, then the ext angles on the same side of the trans are supp.
▸ Given: p || q
▸ Prove: ∠1 and ∠2 are supp.
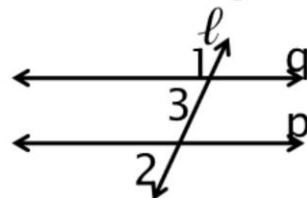
Statements	Reasons

p || q	(given)
m∠1 + m∠3 = 180	(linear pair post)
∠2 ≅ ∠3	(corrs angles post)
m∠2 = m∠3	(def ≅)
m∠1 + m∠2 = 180	(substitution)
∠1 and ∠2 are supp	(def supp)

3.2 Use Parallel Lines and Transversals

- *157 #2–32 even, 36–52 even = 25 total*
- *Extra Credit 160 #2, 6 = +2*

Answers and Quiz

- 3.2 Answers

- 3.2 Quiz

3.3 Prove Lines are Parallel

Corresponding Angles Converse

If 2 lines are cut by trans. so the corrs ∠ are ≅, then the lines are ||.

Alternate Interior Angles Converse

If 2 lines are cut by trans. so the alt int ∠ are ≅, then the lines are ||.

Alternate Exterior Angles Converse

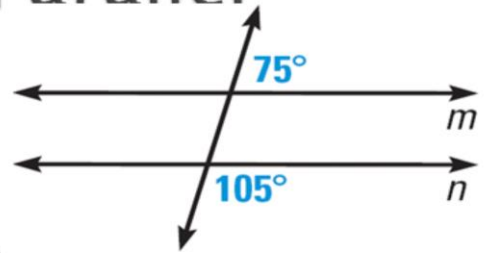If 2 lines are cut by trans. so the alt ext ∠ are ≅, then the lines are ||.

Consecutive Interior Angles Converse
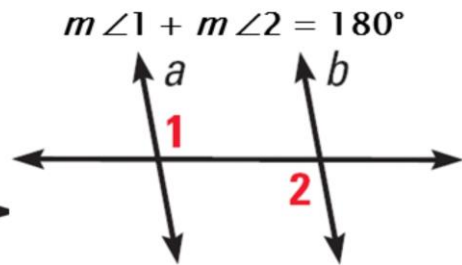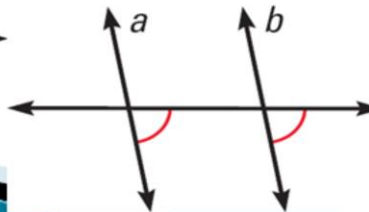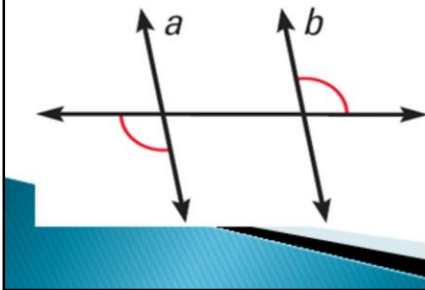
If 2 lines are cut by trans. so the cons int ∠ are supp., then the lines are ||.

3.3 Prove Lines are Parallel

▸ Is there enough information to conclude that m || n?

75°

m

105°

n

▸ Can you prove that the lines are parallel? Explain.

$m \angle 1 + m \angle 2 = 180°$

a   b

a   b

a   b

1

2

Yes, corresponding angles will both be 75°

Yes, alt ext angles converse
Yes, corres angles converse
No, should be $\angle 1 \cong \angle 2$ by alt int angles converse

3.3 Prove Lines are Parallel
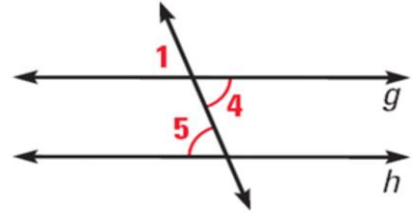
Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.

- Paragraph proofs
  - The proof is written in sentences.
  - Still need to have the statements and reasons.

3.3 Prove Lines are Parallel

▸ Write a paragraph proof to prove that if 2 lines are cut by a trans. so that the alt int ∠s are ≅, then the lines are ||.

▸ Given: ∠4 ≅ ∠5
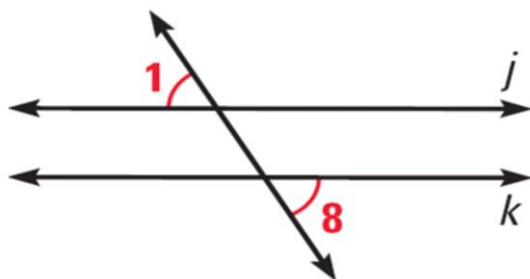▸ Prove: g || h

It is given that ∠4 ≅ ∠5. By the vertical angle congruence theorem, ∠1 ≅ ∠4. Then by the Transitive Property of Congruence, ∠1 ≅ ∠5. So, by the Corresponding Angles Converse, g || h.

3.3 Prove Lines are Parallel

▸ If you use the diagram at the right to prove the Alternate Exterior Angles Converse, what GIVEN and PROVE statements would you use?

165 #2–28 even, 34, 36, 40–54 even = 24 total
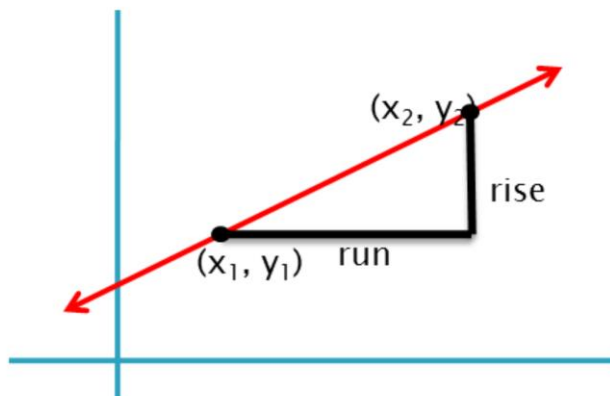
Given: $\angle 1 \cong \angle 8$
Prove: j || k

Answers and Quiz

- 3.3 Answers

- 3.3 Quiz

3.4 Find and Use Slope of Lines

▸ Slope $= \dfrac{\text{rise}}{\text{run}}$

▸ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

3.4 Find and Use Slope of Lines
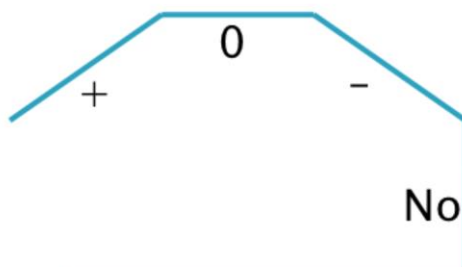
- ▸ Positive Slope
  - ◦ Rises

- ▸ Zero Slope
  - ◦ Horizontal

- ▸ Negative Slope
  - ◦ Falls

- ▸ No Slope (Undefined)
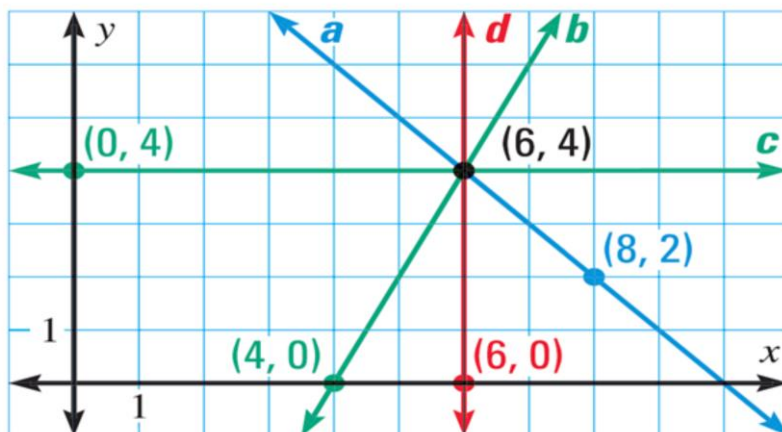 - ◦ Vertical

0

+

−

No

There's **No Slope** to stand on.

3.4 Find and Use Slope of Lines

▸ Find the slope of
  ◦ Line *b*

  ◦ Line *c*

Line b: m = (4 − 0)/(6 − 4) = 4/2 = 2
Line c: m = (4 − 4)/(6 − 0) = 0 / 6 = 0

3.4 Find and Use Slope of Lines

Slopes of Parallel Lines

In a coordinate plane, 2 nonvertical lines are parallel iff they have the same slope.

And, any 2 vertical lines are parallel.

$m_1 = 2; m_2 = 2$

Slopes of Perpendicular Lines

In a coordinate plane, 2 nonvertical lines are perpendicular iff the products of their slopes is −1.

Or, Slopes are negative reciprocals.

And, horizontal lines are perpendicular to vertical lines

$m_1 = 2; m_2 = -\frac{1}{2}$

3.4 Find and Use Slope of Lines

▸ Tell whether the lines are *parallel*, *perpendicular*, or *neither*.
  ◦ Line 1: through (–2, 8) and (2, –4)
  ◦ Line 2: through (–5, 1) and (–2, 2)

  ◦ Line 1: through (–4, –2) and (1, 7)
  ◦ Line 2: through (–1, –4) and (3, 5)

Line 1: (-4 – 8)/(2 – (-2)) → -12/4 → -3
Line 2: (2 – 1)/(-2 – (-5)) → 1/3
Perpendicular

Line 1: (7 – (-2))/(1 – (-4)) → 9/5
Line 2: (5 – (-4))/(3 – (-1)) → 9/4
neither

3.4 Find and Use Slope of Lines

▸ Line q passes through the points (0, 0) and (−4, 5). Line t passes through the points (0, 0) and (−10, 7). Which line is steeper, q or t?

▸ *175 #4–30 even, 34, 36, 40, 44, 46, 48 = 20 total*
▸ *Extra Credit 178 #2, 4 = +2*

m_q = (5 − 0)/(-4 − 0) = 5/-4 = -5/4 = -1.25
m_t = (7 − 0)/(-10 − 0) = 7/-10 = -7/10 = -0.7
Line q is steeper

Answers and Quiz

- 3.4 Answers
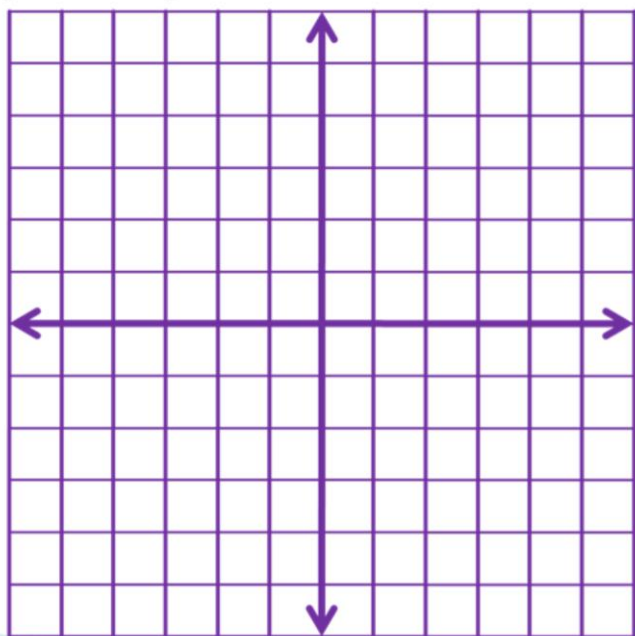
- 3.4 Quiz

3.5 Write and Graph Equations of Lines

▸ Slope-intercept form of a line
  ◦ $y = mx + b$
    • m = slope
    • b = y-intercept

▸ To graph in slope intercept form
  ◦ Plot the y-intercept
  ◦ Move from the y-int the slope to find a couple more points
  ◦ Connect the points with a line

3.5 Write and Graph Equations of Lines
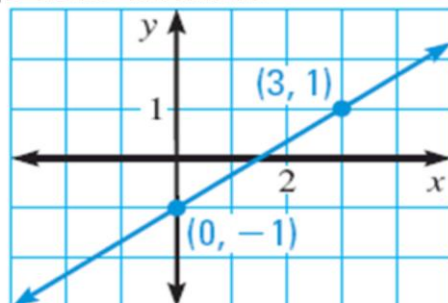
- Graph
  - $y = -2x$

  - $y = x - 3$

3.5 Write and Graph Equations of Lines

- To write equations of lines using slope-intercept form
  - Find the slope
  - Find the y-intercept
    - It is given or,
    - Plug the slope and a point into $y = mx + b$ and solve for b
  - Write the equation of the line by plugging in m and b into $y = mx + b$

3.5 Write and Graph Equations of Lines

▸ Write an equation of the line in the graph

3.5 Write and Graph Equations of Lines

- ▸ Write an equation of the line that passes through (−2, 5) and (1, 2)

3.5 Write and Graph Equations of Lines

- ▸ Write an equation of the line that passes through (1, 5) and is parallel to the line with the equation $y = 3x - 5$.

3.5 Write and Graph Equations of Lines

▸ Standard Form
  ◦ $Ax + By = C$
    · A, B, and C are integers

▸ To graph
  ◦ Find the x- and y-intercepts by letting the other variable = 0
  ◦ Plot the two points
  ◦ Draw a line through the two points

x-intercept:
$Ax + B(0) = C$
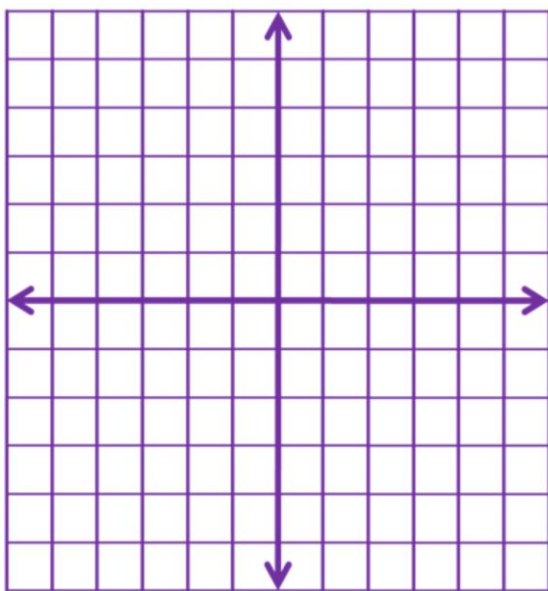$Ax = C$
$x = C/A$

Y-intercept:
$A(0) + By = C$
$By = C$
$y = C/B$

3.5 Write and Graph Equations of Lines

- Graph
  - $2x + 5y = 10$

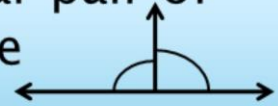- *184 #2–12 even, 16–26 even, 30–36 even, 40, 44, 46, 60, 62, 68–74 even = 25 total*
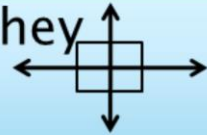
Answers and Quiz

- 3.5 Answers

- 3.5 Quiz
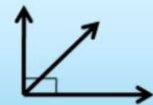
3.6 Prove Theorems About Perpendicular Lines

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

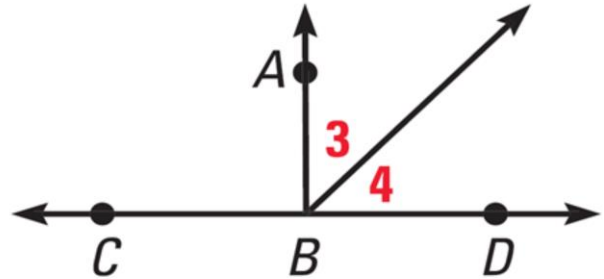If two lines are perpendicular, then they intersect to form four right angles.

If two sides of two adjacent angles are perpendicular, then the angles are complementary.
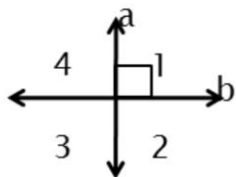
3.6 Prove Theorems About Perpendicular Lines

▸ Given that ∠ABC ≅ ∠ABD, what can you conclude about ∠3 and ∠4?

3.6 Prove Theorems About Perpendicular Lines

▸ Prove that if two lines are perpendicular, then they intersect to form four right angles.
▸ Given: a ⊥ b
▸ Prove: ∠1, ∠2, ∠3, ∠4 are rt ∠s.

Statements	Reasons

a ⊥ b	(given)
∠1 is rt angle	(def ⊥ lines)
m∠1 = 90°	(def rt angle)
m∠1 + m∠2 = 180	(linear pair postulate)
90 + m∠2 = 180	(substitution)
m∠2 = 90	(subtraction)
∠2 is rt angle	(def rt angle)
∠3 ≅ ∠1, ∠4 ≅ ∠2	(vertical angles are ≅)
m∠3 = m∠1, m∠4 = m∠2	(def ≅)
m∠3 = 90, m∠4 = 90	(substitution)
∠3 is rt ∠, ∠4 is rt ∠	(def rt ∠)

3.6 Prove Theorems About Perpendicular Lines

Perpendicular Transversal Theorem

If a trans. is ⊥ to 1 of 2 || lines, then it is ⊥ to the other.
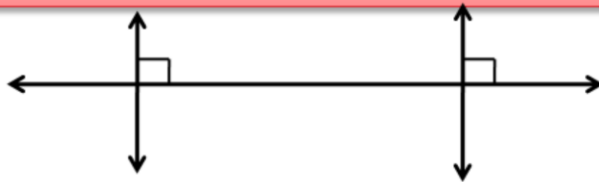
Lines ⊥ to a Transversal Theorem
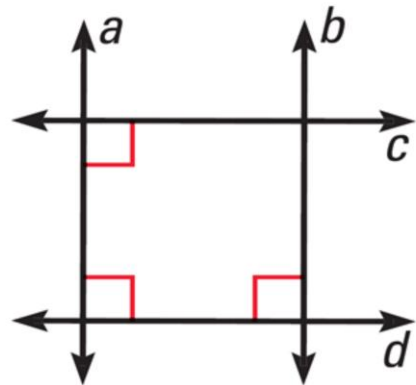
In a plane, if 2 lines are ⊥ to the same line, then they are || to each other.
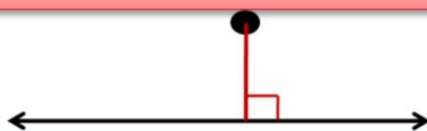
3.6 Prove Theorems About Perpendicular Lines

▸ Is b || a?

▸ Is b ⊥ c?

Yes, lines perpendicular to transversal theorem

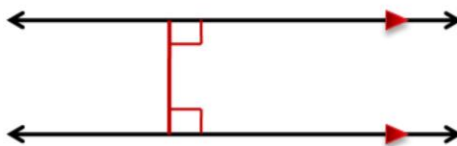Yes, c || d by the lines ⊥ to trans theorem; b ⊥ c by the ⊥ trans theorem

3.6 Prove Theorems About Perpendicular Lines

Distance

From point to line: length of segment from point and $\perp$ to line
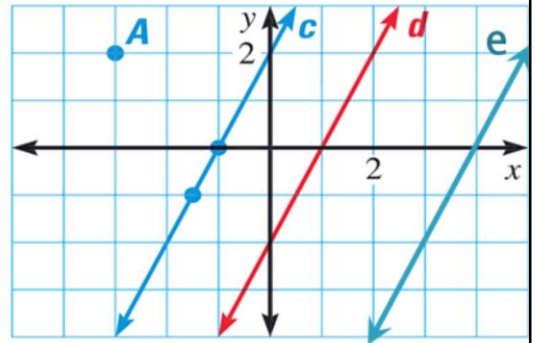
Between two $\parallel$ lines: length of segment $\perp$ to both lines

3.6 Prove Theorems About Perpendicular Lines

▸ What is the distance from point A to line d?

▸ What is the distance from line c to line e?

Slope of line c = 2 (rise = 2, run = 1)

Slope of $\perp$ line = -1/2

Follow slope from A(-3, 2) to line cd; intersection at (1, 0)

Calculate distance $\sqrt{\left(1 - (-3)\right)^2 + (0 - 2)^2} = \sqrt{4^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5} = 4.47$

Point on line c: (0, 2)

Follow slope from (0, 2) to line e

Point of intersection (4, 0)

Distance = $\sqrt{(4 - 0)^2 + (0 - 2)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} = 4.47$

3.6 Prove Theorems About Perpendicular Lines

- *194 #2–10 even, 14–26 even, 30–46 even = 21 total*
- *Extra Credit 197 #2, 8 = +2*
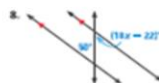
Answers and Quiz

‣ 3.6 Answers

‣ 3.6 Quiz

3.Review

▸ *206 #1–25*
  = 25 total

Classify the pairs of angles as *corresponding, alternate interior, alternate exterior,* or *consecutive interior.*

1. $\angle 1$ and $\angle 8$ 2. $\angle 2$ and $\angle 6$ 3. $\angle 3$ and $\angle 5$

4. $\angle 4$ and $\angle 5$ 5. $\angle 3$ and $\angle 7$ 6. $\angle 3$ and $\angle 6$

Find the value of x.

7.

8.

9.

Find the value of x that makes $m \parallel n$.

10.

11.

12.

Find the slope of the line that passes through the points.

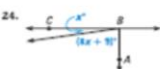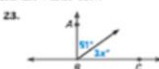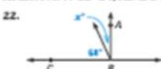13. $(3, -1), (3, 4)$ 14. $(2, 7), (-1, -3)$ 15. $(0, 5), (-6, 12)$

Write an equation of the line that passes through the given point P and has the given slope m.

16. $P(-2, 4), m = 3$ 17. $P(7, 12), m = -0.2$ 18. $P(3, 5), m = -8$

Write an equation of the line that passes through point P and is perpendicular to the line with the given equation.

19. $P(1, 3), y = 2x - 1$ 20. $P(0, 2), y = -x + 3$ 21. $P(2, -3), x - y = 4$

In Exercises 22–24, $\overline{AB} \perp \overline{BC}$. Find the value of x.

22.

23.

24.

25. **RENTAL COSTS** The graph at the right models the cost of renting a moving van. Write an equation of the line. Then find the cost of renting the van for a 100 mile trip.

48